Feedback Considered Beneficial: Exploring Frequency Diversity in Full-duplex Rateless Codes

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***Abstract*—Recently, rateless codes have displayed a promising paradigm for rate adaptation. They enable the sender to transmit packets with a fixed data rate, and the receiver to decode the packets with a data rate comparable to its channel condition. Therefore, rateless codes can achieve higher wireless throughput than the fixed rate codes, especially over time-varying channels. However, as wireless multicarrier techniques become essential technologies in current communication systems, the design of rateless code exposes a critical problem. That is, it is not able to leverage frequency diversity introduced by multi-carrier techniques, which will degrade the achievable data rate. Motivated by this, we propose a Full Duplex Rateless (FDR) code in *frequency domain*. It aims to take advantage of the frequency diversity and fine-grained feedback to achieve the "most appropriate" data rate on every single subcarrier in a multicarrier communication system. Extensive simulations have been conducted to verify the effective of FDR code, and the results show that FDR code achieves up to 6.5× data rate over the fixed-rate LDPC codes, and 2.3× data rate over rateless Spinal code.**

I. INTRODUCTION

Wireless techniques have gained the tremendous popularity in recent years. Accordingly, the demand for high throughput and high data rate wireless communication systems increases dramatically. To meet this demand, rateless codes have gained renewed strong interest. In traditional fixed-rate code (e.g., Low- Density Parity-Check (LDPC) adopted by 802.11), the sender needs to predict or obtain the channel condition at the receiver side, and then picks different combinations of coding and modulation schemes to adapt the transmission data rate. On the contrary, rateless codes do not require any estimation or information about the channel. The sender generates a potentially limitless number of rateless symbols from a fixed number of message symbols, and the receiver continues collecting the rateless symbols until all the message symbols are successfully decoded. Strider [1] and Spinal [2] are two representatives. Strider uses a layered approach to generate linear combinations of symbols in a rateless manner. Its good empirical performance verifies the feasibility and effectiveness of the rateless codes. However, this layered approach relies on the existing fixed-rate code and symbol set, and the structure becomes complex as the number of layers increases. Meanwhile, Spinal code utilizes hash function and random number generator to produce rateless symbols, which simplifies the design compared to Strider.

However, as wireless multicarrier techniques become prevailing, the design of rateless codes faces new challenges and

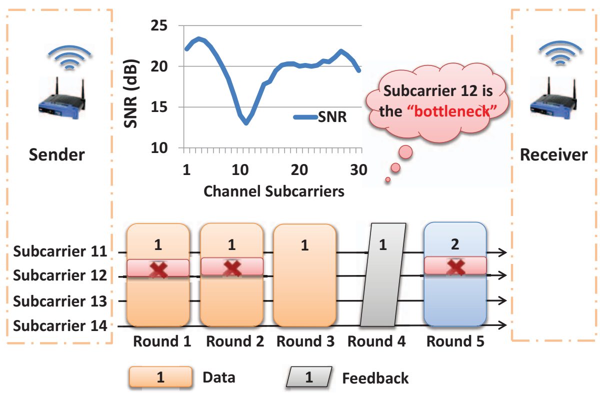

Fig. 1: The state-of-art rateless codes cannot leverage frequency diversity. Subcarrier 12 becomes the "bottleneck", which significantly restricts the overall data rate.

opportunities. In a multicarrier communication system, such as OFDM (Orthogonal Frequency Division Multiplexing) based system, a frequency-selective wideband channel is transformed into several flat-fading narrowband subchannels supported by subcarriers. Therefore, it has the ability to fight against multipath fading and increase the spectral efficiency. Meanwhile, the signals carried by different subcarriers experience quite diverse transmission quality, which introduces a significant frequency diversity across the entire channel. Such frequency diversity also leads to various "decodability" across different subcarriers. The term "decodability" refers to the probability that a receiver can correctly decode the received symbol on a particular subcarrier, which highly relies on the channel condition on that subcarrier. Previous rateless codes treat all subcarriers as the same, and do not take decodability diversity into consideration. Therefore, the subcarrier with higher decodability will be wasted, and the one with lower decodability will become the "bottleneck". Just as the "Cannikin Law" goes, how much water a bucket can contain is determined by the shortest board. This indiscrimination significantly degrades the overall throughput when the frequency diversity is strong.

We argue that the state-of-art rateless codes only focus on the receiver side, where feedbacks to the sender is too coarse. Thus the sender cannot obtain fine-grained information to leverage the frequency diversity, like the decoding status or decodability of each subcarrier. With proper and fine-

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grained feedback, the sender can do better adjustment for rate adaptation together with the receiver, and thus enhance the data rate and overall throughput. Motivated by this, we propose a Full Duplex Rateless (FDR) code in OFDM-based systems to obtain a higher and more appropriate date rate on each subcarrier. To be specific, OFDM modulation provides fine-grained and cost-efficient feedbacks(e.g., decoding statues on each subcarrier), while full duplex paradigm for wireless transceiver [3] enables the sender and receiver to transmit data packets and feedbacks simultaneously, which guarantees the latest feedbacks to the sender.

To the best of knowledge, FDR code is the first design in the literature that utilizes full-duplex paradigm in OFDM based system to utilize frequency diversity for better rateless coding. Our experiments show that compared to fixed-rate LDPC codes and rateless Spinal code, FDR code improves the data rate by up to 6.5× and 2.3× respectively.

II. Full-duplex Rateless coding (FDR)

In this section, we describe the overall architecture of a Full Duplex Rateless (FDR) code enabled communication system, which aims to harness frequency diversity and fine-grained feedback to obtain a capacity-approaching code.

A. FDR Code Overview

In a FDR code enabled system, the sender and receiver are both equipped with full duplex transceivers. Spinal code and OFDM are adopted as the basic encoding and modulation scheme. Obviously, other type of rateless codes can also be integrated into FDR. Here Spinal code is chosen because it is a good capacity-approaching code with a simple structure for illustration. Before we get to the detailed description of FDR code, two necessary assumptions are listed as follows: 1) Currently we only consider point to point transmission, e.g., transmission between a sender-receiver pair, as in Spinal [2]. 2) We assume the channel is slowly time-varying, e.g., remains stable within a small number of transmission rounds, then 802.11n preamble can be used periodically for synchronization to reduce the MAC layer overhead.

The fundamental idea of FDR code is to *leverage frequency diversity and fine-grained feedback to achieve a higher data rate and fully utilize the channel capacity*. The basic idea of FDR is simple and efficient, yet there remains several challenges for implementation. First, rateless codes encode symbols in a correlated way among subcarriers. If we utilize frequency diversity to transmit and decode symbols individually on each subcarrier, the correlation should be broken down carefully. Second, feedbacks should be designed efficiently to provide latest and accurate decoding status. Third, the vacated subcarriers with high decodability should be utilized efficiently to improve the overall data rate. We see how FDR code addresses these challenges in the following subsections.

B. FDR Encoding and Decoding

FDR encoder and decoder abstract the basic idea of Spinal code for illustration. To enable decoding on a single subcarrier,

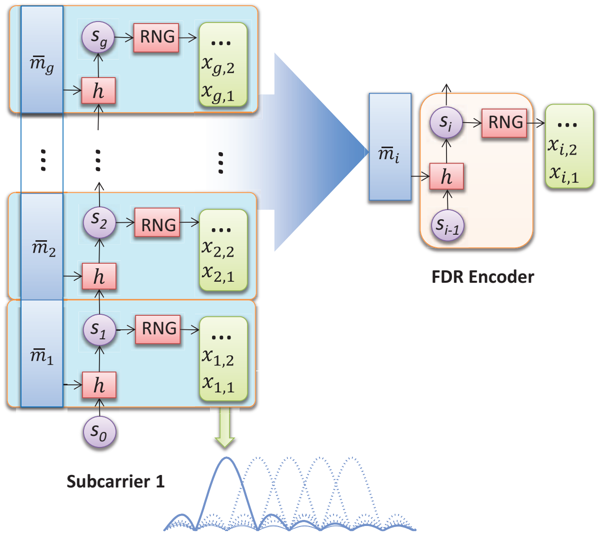

Fig. 2: FDR encoder breaks up the correlation of a message block M among subcarriers, and maintains such correlation within a single subcarrier

we redesign the structure of Spinal, and break down the correlation among subcarriers to achieve a frequency independent rateless code. Then the rateless symbols can be decoded independently on each subcarrier according to the decodability diversity. Specifically, we assign one unique subcarrier to a particular message block M. Instead of transmitting one block at a time, we transmit multiple blocks on all the subcarriers concurrently. In this way, the correlation of rateless symbols among subcarriers is broken down, and is only maintained within a single subcarrier.

The detailed encoding process is as follows: the encoder first divides a packet into multiple message blocks. Each message block M has a length of $(n-16)$. A 16-bit CRC is computed and inserted at the end of M to construct a link-layer frame. If there are N subcarriers, then we take N message blocks as a *batch*, and encode them in parallel on each subcarrier. As shown in Fig. 2, FDR encoder consists of one hash function h and one Random Number Generator (RNG). Every k message bits $\vec{m}_i$ ($\{0,1\}^k$) are taken from block M and encoded into one rateless symbol $x_{i,j}$, just as in Spinal [2]. The spine value s_i is constructed by sequential hashing h, and each s_i is seeded to RNG to generate sequences of rateless symbols $x_{i,j}$ for message bits $\vec{m}_i$.

During the decoding process, FDR simply applies bubble decoder [2] to every single subcarrier instead of the whole channel. To maintain the transmission schedule on each subcarrier, sender and receiver first construct a *decoding table*(which message block is transmitted on which subcarrier in current round). They also construct a set *unfinished* to store the undecoded message blocks in a batch. The transmission schedule is synchronized between sender and receiver since they apply the same block allocation strategy as described in Sec. II-C. Upon receiving a symbol on a certain subcarrier, the receiver will look up the table and find out which message

Algorithm 1 FDR Decoder.

for i = 1 **to** N **do**
  Match the received symbol with the right message block according to the transmission schedule in the *decoding table*.
end for
for j = 1 **to** the number of unsuccessful blocks **do**
  Conduct bubble decoding.
  if the j^{th} block is decoded successfully **then**
    Delete M_j from *unfinished*.
  end if
end for

block the symbol belongs to. Afterwards, bubble decoding is conducted on that subcarrier. If this message block has been successfully decoded, the receiver will delete it in set *unfinished*. Algorithm 1 depicts the pseudo-code of the decoding procedure in FDR.

C. Fine-grained Feedback and Block Allocation

Fine-grained feedback is an essential component in FDR. It provides the latest information about the decoding status on each subcarrier. With such information, sender is able to utilize the decoding diversity to obtain a capacity-approaching codes. Since we adopt full duplex transceiver in FDR, in each transmission round, sender and receiver are able to transmitting data symbols while exchanging feedback. As shown in Fig. 3, starting from round 2, the receiver is capable of providing a feedback to the sender. This feedback is an OFDM symbol, where each subcarrier is modulated with Binary Phase Shift Keying (BPSK) signal "1" or "0". "1" represents that the message block on that subcarrier has been successfully decoded in this round, and "0" represents that the message block has not been decoded yet.

One critical problem is that, the sender can only obtain the feedback of the previous transmission round, e.g., in round 2, the sender obtains the decoding status of the symbols transmitted in round 1. This delayed feedback makes the sender unable to decide what it should transmit in the current round (e.g., round 2). To address this problem, we partition the transmission rounds into *odd rounds* and *even rounds*, where *odd rounds* includes transmission rounds with odd time index (e.g., round 1 and 3), and *even rounds* includes transmission rounds with even time index (e.g., round 1 and 3). In *odd rounds* we transmit a batch of message blocks, and in *even rounds* we transmit another batch of message blocks. In this way we allow one round interval between the transmission of the same batch of blocks, which enable the sender to obtain the latest decoding status. However, another problem arises. As the transmission of two batches of message blocks are interleaved, each of the transmission time is expected to be doubled, which enlarges the transmission delay. To minimize this delay, we use two subcarriers to transmit one rateless symbol. As shown in Fig. 3, subcarrier 1 and 3 are used to transmit message block 1, and subcarrier 2 and 4 are used to transmit message block 2.

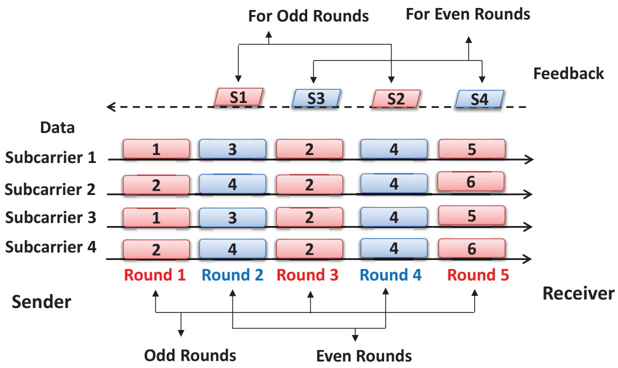

Fig. 3: Fine-grained feedback and transmission schedule in FDR, where *odd* rounds and *even* rounds are used to transmit different batches of message blocks

In this way, the delay cost in time domain can be compensated from the frequency domain.

When a sender obtains the current decoding status and vacates the subcarrier on which the message block has already been decoded, it will utilize block allocation algorithm to reassign a message block to that subcarrier. Here we propose a simple *Cyclic Allocation Algorithm* for illustration. To be specific, the sender and receiver first number all the subcarriers in ascending order starting from index 1 to N. Two batches of message blocks are allocated to the subcarriers from 1 to N one after another, each message block with 2 subcarriers. This schedule is written into the *decoding table* at both the sender and receiver side for records. Upon receiving a "1" on a certain subcarrier, the message block on that subcarrier is expected to be decoded successfully, so the sender deletes it from the *decoding table*. Afterwards, it vacates the corresponding subcarriers, and allocate them to the first message block picking from the *decoding table*. This allocation is conducted cyclically until all the vacated subcarriers are occupied. Since the sender and receiver have the same information, it is easy for them to keep pace in the transmission schedule.

III. Performance Evaluation

In this section, we evaluate the performance of FDR through extensive interconnected simulations written in C++ and Matlab. Two state-of-art codes are used for comparison: Fixed-rate LDPC codes and rateless Spinal code. For LDPC codes, the combinations of channel coding and modulation schemes are the same as in 802.11n [4], and the rate adaptation algorithm is SoftRate [5]. We plot the highest rate achieved by all the possible combinations of channel coding and modulation schemes for each SNR value, to demonstrate the best envelope of LDPC codes. As for Spinal code, we follow the configurations indicated in [2]. Every pass of 32 rateless symbols are punctured into 8 subpasses, each with 4 rateless symbols. Since OFDM modulation could transmit more than one pass of rateless symbols at a time (e.g., 48 rateless symbols at a time if 64-point FFT is used), the data rate will drop dramatically from $8k$ to $\frac{n}{48L}$, where k and n are the group size and message

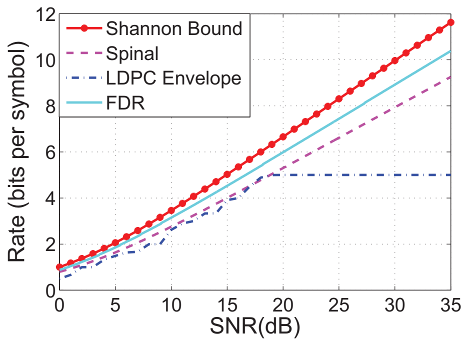

Fig. 4: The achieved data rate by FDR, LDPC and Spinal under a AWGN channel

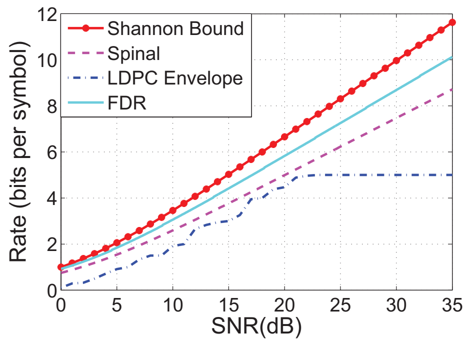

Fig. 5: The achieved data rate by FDR, LDPC and Spinal under a flat fading channel

TABLE I: Simulation Parameters for LDPC, Spinal and FDR

Code	Parameter	Value
LDPC	Message size n	648 bits
Spinal	Message size n	128 bits
	Group size k	4 bits
	Beam size B	256
FDR	Message size n	128 bits
	Group size k	4 bits
	Survivor size V	256

block size respectively, and L is the number of passes needed for decoding. To enhance the performance of Spinal, we use a similar parallel transmission structure as in FDR, where 12 message blocks are multiplexed in one OFDM symbol. Then for each message block, 4 rateless symbols are transmitted per OFDM symbol, resulting a data rate of $8k/L$. We implement FDR as described in Sec. II. To reduce the overhead of feedback, we puncture every 32 rateless symbols per round in a message block into 4 subrounds, each with 8 rateless symbols. The detailed parameters are shown in Table I.

In the following simulations, we use a sender-receiver pair topology, where a sender keeps transmitting packets to a receiver using LDPC, Spinal and FDR respectively. Each run of a simulation transfers 2500 packets with a packet length of 1460 bytes. For each value of a SNR, the simulation is repeated 10 times. We choose the MAC layer received data rate as our basic performance metric, which is calculated by: multiplying the bits per symbol with channel bandwidth, and subtracting MAC layer overheads. The channel bandwidth is 20MHz, with 64-point FFT modulation. 48 subcarriers are used for data transmission, and 4 subcarriers serves as pilots. We measure the SNR range from 0dB to 35dB with 1dB interval. We use three channel models to verify the effectiveness of FDR, including AWGN channel, flat fading channel and frequency selective channel. Without loss of generality, we use Shannon Bound in AWGN channel for illustrations in both flat fading channel and frequency selective channel, since we assume it is the upper bound in all the channel models.

A. Performance over AWGN Channel

We first evaluate the performance of FDR over a AWGN channel compared with LDPC and Spinal. Ideally, there is no fading or frequency selectivity. Therefore, FDR is expected to achieve a comparable performance with Spinal. Fig. 4 shows the achieved data rate as a function of SNR. When exceeds a certain threshold, LDPC is bounded by a certain data rate (e.g., 5 bits/symbol achieved by 64QAM with 5/6 coderate), making it unable to utilize higher SNR to achieve higher data rate. Conversely, both FDR and Spinal are expected to achieve date rates that are very close to Shannon Bound across the entire SNR range. That is because when SNR is relatively high, rateless codes are able to use less time for decoding than fixed rate codes. While when SNR is relatively low, rateless codes still take this advantage as temporal noise varies over time. Thus FDR achieves up to 60% date rate gain over LDPC codes. FDR further outperforms Spinal by between 5% (0dB to 10dB) and 10% (above 20dB). These gains also benefit from the noise fluctuation on different subcarriers, making some message blocks to be decoded earlier than others. FDR just takes these opportunities to achieve a higher data rate.

B. Performance over Flat Fading Channel

Next, we evaluate the performance of FDR over a flat fading channel compared with LDPC and Spinal. We choose the IEEE 802.11 channel model from [6], which is a Rayleigh fading environment with little frequency selectivity. The performance comparisons among FDR, LDPC and Spinal are shown in Fig. **??**. Not surprisingly, LDPC achieves lower date rate than in AWGN channel due to fading, and its data rate is still bounded at 5 bits/symbol when SNR exceeds a threshold (e.g., 19dB). Spinal also performs worse than in AWGN channel due to slight frequency selectivity, which results in different decodability among various subcarriers. Symbols on subcarriers with lower decodability need longer time for decoding. So Spinal has to keep transmitting more rateless symbols on all the subcarriers, even though not all of them are helpful. Meanwhile, FDR can efficiently utilize frequency diversity to avoid transmitting the message blocks that have been already decoded. The vacated subcarriers are then used to transmit other undecoded message blocks. Therefore, it can approach higher and more appropriate data rate for each subcarrier. The results show that FDR outperforms Spinal by up to 45%, and achieves an average gain of 15% under a typical SNR working range(e.g., 20dB to 30dB).

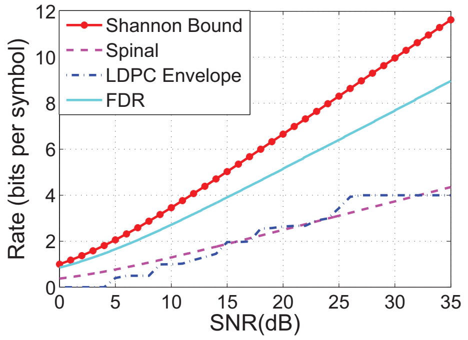

Fig. 6: The achieved data rate by FDR, LDPC and Spinal under a frequency-selective channel

C. Performance over Frequency Selective Channel

It is expected that FDR can achieve the best performance over frequency selective channel, since it aims to leverage frequency diversity to achieve higher data rate. We generate a frequency selective channel according to the model in [6], with a maximum RMS delay δ_t of 80ns and paths of 16. For each path, the amplitude and phase are random variables following Rayleigh and Uniform distribution respectively. Fig. **??** depicts the achieved data rate as a function of SNR for LDPC, Spinal and FDR. We can see that both LDPC and FDR perform badly due to deep fading and frequency selectivity, even SNR is very high. As we discussed before, the symbols on the subcarrier with low decodability become "bottleneck" in Spinal, which significantly restricts the overall data rate. Conversely, FDR achieves a desirable date rate and approaches the Shannon bound across the whole SNR range. It outperforms Spinal by up to 115%, and achieves an average gain of 100% under a typical SNR working range. Moreover, under some channel conditions, LDPC achieves a little bit higher data rate than spinal. This mainly benefits from the Forward Error Correcting (FEC) adopted by LDPC, which allows LDPC to correct the missing part of the received symbols even not all them an be received successfully. As for spinal, since some rateless symbols are essential for decoding (e.g., tail symbol), missing such symbols will definitely degrade its performance.

IV. Related Work

Fixed-rate codes and modulation schemes, such as convolutional codes [7] and LDPC codes [8], have been adopted for a long time by current 802.11 and a variety of cellular wireless standards. However, they requires explicit rate adaptation policies at the sender side, which is claimed to be either inaccurate or costly. Rateless codes, such as LT codes and Raptor codes, have displayed a desirable paradigm for rate adaptation recently. In [1], a layered approach termed Strider is proposed to generate linear combinations of symbols in a rateless manner. While in [2], hash function and random number generator is utilized by Spinal code, which simplifies the design compared with layered approach. These codes work well in single carrier communication systems, making fine-grained feedback become not quite necessary. However, in multicarrier communication system, fine-grained feedback help the sender utilize frequency diversity to achieve better performance. Harnessing decodability diversity, rate adaptation can be performed better, and achieves a higher, more appropriate and fine-grained data rate on each subcarrier in a multicarrier communication system.

V. Conclusion

In this paper, we propose a novel Full-Duplex Rateless code (FDR) in *frequency domain* to harness frequency diversity and fine-grained feedback for rate adaptation. In a multicarrier communication system, state-of-art fixed-rate codes and rateless code cannot utilize frequency diversity to achieve a capacity approaching code. By utilizing fine-grained feedbacks, FDR can "smartly" decode the symbols on each subcarrier according to the channel condition. Extensive simulation results show that FDR achieves up to 6.5× performance gain than 802.11 LDPC codes, and 2.3× performance gain than rateless Spinal codes over a frequency selective channel.

In next stage, we propose to validate FDR on SDR platform [9] [10], and integrate more functionalities, including error correction and estimating into FDR to benefit more applications.

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